

# An Online Learning-based Metaheuristic for Solving Combinatorial Optimization Problems

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**Keywords:** *Combinatorial optimization, Metaheuristics, Machine learning, Hybridization.*

## 1 Introduction

Combinatorial optimization problems (COPs) are a complex class of optimization problems with discrete decision variables and a finite search space. They have a wide application in many real-world problems, including transportation, scheduling, network design, assignment, and so on. Many COPs belong to the NP-Hard class of problems, which require exponential time to be solved to optimality [1]. For these problems, metaheuristics (MHs) provide acceptable solutions in reasonable computation times, and are often good substitutes for exact algorithms.

Machine learning (ML) techniques are also good approaches for solving COPs [2]. In this regard, the hybridization of ML techniques with MHs is an emerging research field that has attracted numerous researchers in recent years [3, 4, 5]. ML techniques can be used to improve the performance of MHs, particularly for solving complex COPs. In the hybrid framework, ML techniques are used to extract knowledge from available data, and inject it into MHs, with the aim of reducing computational time, and improving solutions quality.

The goal of this contribution is twofold: 1) *Proposing a state of the art review on hybridization methods between MHs and ML*; and 2) *Introducing the concept of a novel approach focused on online learning in population-based MHs*.

## 2 Literature Review

In this section, we briefly review the main directions for hybridization of MHs and ML techniques. This hybridization can happen at different levels (*i.e.*, local and global), and can be done either before the search starts (offline learning) or during the search (online learning) [4].

At a local hybridization level, ML is applied to specific parts of the MHs such as solutions initialization, parameters tuning, objective function approximation, and/or population management [3]. ML can either 1) improve the quality of the obtained results (*e.g.*, by incorporating knowledge into the MHs components in order to search the solution space intelligently, or tune parameters autonomously in an a priori manner, or dynamically during its search process); and/or 2) speed up the search process, for instance by approximating the objective function value especially when its computing cost is high.

At a global hybridization level, ML can be used to make some approximations of algorithmic decisions that would normally require a high computational time. Hyper-heuristics, algorithm selection, and cooperative strategy are the main forms of hybridization in this category, that integrate learning components in order to improve problem solving [3]. Hyper-heuristics are high-level strategies that select or generate low-level, problem-specific, heuristics for solving a particular problem instance (or a class of instances). Reinforcement learning is a common approach in hyper-heuristics literature for selecting the low-level heuristics based on an online

learning strategy [6]. Algorithm selection chooses one or more algorithms that perform best for a given problem instance, based on the instance features, using meta-learning approaches [7]. Cooperative strategy makes a set of MHs work together in a parallel/sequential manner, in order to exploit advantages of each algorithm and better explore the search space. This cooperation is usually modeled as a multi-agent system with a coordinator agent, which uses the knowledge extracted from individual performance of each MH to guide them jointly [8].

### 3 Proposed Approach

Most of the approaches in the literature focus on offline hybridization, that requires high computational effort for generating training data and calculating instance features. In this study, we focus on online learning in population-based MHs. Indeed, a population of solutions being evolved through a number of iterations carries a huge amount of information, which can be used to incrementally train a ML algorithm. We first identify a set of features that well characterize the good solutions obtained in previous populations, and use them to incrementally train a ML technique. Afterwards, the extracted knowledge (*e.g.*, rules) is employed to help the MH explore promising areas (or avoid unpromising ones) of the solution space. This leads to better solutions generation for the subsequent populations, as well as acceleration of the process, due to intelligent search of the solution space. In order to test the applicability and performance of the proposed hybrid intelligent algorithm over state-of-the-art algorithms, we have focused on the traveling salesman problem, which is a well-known NP-Hard COP with wide applications in real-world problems.

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