

Constraint Programming Approaches for the RCPSP with Routing

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Introduction

The RCPSPR (Resource Constrained Project Scheduling Problem with Routing) as the RCPSP ((Resource Constrained Project Scheduling Problem)) is composed a set of activities, $V = \{A_0, \dots, A_{n+1}\}$, with durations, $p = (p_0, \dots, p_{n+1})$, where n is the number of non-dummy activities. All this activities define the project. Two dummy activities, A_0 and A_{n+1} , such that $p_0 = p_{n+1} = 0$, model the “project start” and the “project end”, respectively. The set of non-dummy activities is identified by $A = \{A_1, \dots, A_n\}$. The activities are linked by two kinds of constraints, the precedence constraints (one activity j cannot start before all its predecessors have been achieved) and the resource constraints induced by the resource exchanges (an activity requires resources to be achieved). A schedule of the RCPSP can be represented as a vector of activity start times, $S = (S_0, \dots, S_{n+1})$, where $S_i \in \mathbb{N}$, with the associated vector of activity completion times, $C = (C_0, \dots, C_{n+1})$. The precedence graph is denoted $G = (V, E)$, where nodes in V are activities and edges in E are precedence relations. For each activity $A_i \in V$, all outgoing arcs $(A_i, A_j) \in E$ are weighted by its duration p_i . If there are arcs $(A_i, A_j) \in E$, then $C_i = S_i + p_i \leq S_j$ since activity A_j has to be scheduled after activity A_i . Each activity requires some amount of renewable resources. The number of project resources is denoted as q and the set of resource is $R = \{R_1, \dots, R_q\}$. The activity resource requirement $b_{ik} \in \mathbb{N}$ means that activity A_i requires $b_{ik} \leq B_k$ resource units of resource k during its execution such that B_k denotes the availability of resource k .

The RCPSPR also encompasses a routing problem since the resources should be transported (transferred) between the activities by one or several vehicles. The RCPSPR is defined by a set of vehicles $T = \{T_1, \dots, T_v\}$ sorted in descending order of capacity $c_u, u \in T$ with a loaded transportation time t_{ij} from activity A_i to A_j vehicle-independent and vehicle load-independent. The objective of the RCPSPR is to minimize the total duration of the project, given by the starting time of the dummy activity A_{n+1} , corresponding to the makespan C_{max} .

The linear resolution of the RCPSPR remains intractable since it required a large number of binary variable with disjunctive constraints for several NP hard problems. The idea we promote consists in taking advantages of classical RCPSP modeling into a global iterative process to obtain solutions in reasonable time first and to create the first step of an iterative resolution for the RCPSPR.

RCPSPR resolution via Constraint Programming Approach

Several formulations were introduced for the RCPSP [1, 7, 8, 3] and more recently, with a flow formulation [2] that defines a solution of the RCPSP using an activity-on-node (AON)-flow network defining a graph. In this graph, there is a vertex for each activity, and the resource arcs represents the number of units of the resource directly transferred between two activities. This problem, like numerous routing and/or scheduling problems encompass a lot of constraints. The idea is based on sub-problem definitions to solve the integrated problem using constraint optimization approach [4]. Constraint programming have been proven to tackle efficiently these problems by providing one solution in short computation time. We propose to take advantages of the Resource Constrained Project Scheduling Problem flow formulation and constraint programming, to define an iterative search process based on:

1. The definition of a flow for the RCPSP
2. The transformation of one RCPSP solution into a solution of the RCPSPR in three steps
 - 2.1. Definition of date compliant RCPSP solution with the flow ;
 - 2.2. Addition of transportation time as the RCPSP with Transfer Time [5];
 - 2.3. Resolution of the transportation problem to obtain a solution of the RCPSPR.

Our contribution is the definition of four constraint programming formulations dedicated to each step of the iterative process that has been proved to be efficient in several instances, introduced in 2018 [6], with less than 10 activities and up to 20 transport operations. For the implementation, we use the CP solver Choco 4.10.0. Numerical experiments have also been conducted with a linear formulation with CPLEX in order to prove the effectiveness of the approach developed in this paper.

Concluding remarks

This work is the first step into the definition of a new trend of models for the RCPSPR based on constraint programming technics. These promising results, pushes us to test this method on larger instances (with 30 activities and nearly 50 transport operations). Our research is also directed into the definition of a global scheme to investigate efficiently the solutions search space with constraint programming and heuristics.

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