Benders decomposition algorithm for a generalized Resource Leveling Problem

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1 Introduction

We study a generalized formulation of a classical Resource Leveling Problem (RLP) with overload cost minimisation problem. This problem involves a project with activities and resources utilized to implement these activities with some requirements and constraints. This problem is proved to be NP-hard in the strong sense [5]. In contrast to RCPSP, RLP involves minimization of resource utilization fluctuations or extra resource cost (overload) during some fixed project duration. Basic objective functions were considered, for example, by Rieck and Zimmermann [6]. RLP models were enriched with several features. Firstly, activities were allowed to have different intensity in time periods and various duration. This formulation was studied by Hans [3] in his thesis with branch and price algorithm, later Kis [4] considered a dual branch and cut method with good results. Generalized precedence relations were formulated by Bianco et al. [2] with solution approach. Secondly, Baydoun et al. [1] studied the model with continuous duration allowing to provide start and end moments and special conditions of precedence relations such as allowed overlapping.

2 Problem formulation and solution approach

Our generalized formulation is based on models of Bianco et al. [2] and Baydoun et. al [1], with variable and continuous activity duration (and continuous starts and ends). The objective function is total extra resource usage cost under the constraint of fixed project deadline. Planning horizon is divided into periods of equal length. A set of resources is given, with available workload amount provided for each period and extra resource workload cost. There is a set of activities which must be executed without preemption, each activity requires different workload amount implemented by different resources. Activities also have upper and lower limit for the workload amount allowed to be assigned in period, and there are precedence relations. In contrast to formulations presented in literature, in our model activity fraction in period decision variable is disaggregated on the resource set R. It allows to allocate resources independently and provides flexible resource distribution. So instead of one fraction decision variable per activity and period , we have $|R|$ decision variables for each resource type.

We use Benders decomposition approach for the model MILP problem, implemented with CPLEX 12.8. In solution process we iteratively solve master and dual of the subproblem, master provides partial solution to the dual subproblem and the dual subproblem is used to construct new cuts for the master. Process is repeated until there are equal values of lower and upper bound. In our case master has no objective function and constructs a feasible schedule. Resource allocation is implemented in subproblem. We also apply several features to accelerate basic algorithm :

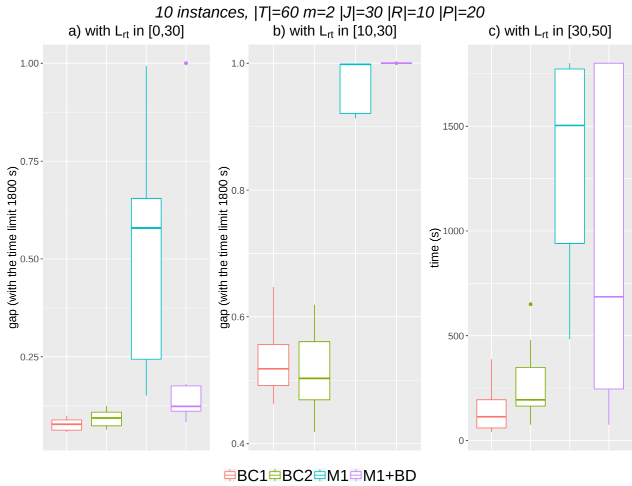

FIG. 1 – Time box plots for instances with 30 activities

— master model lower bound ;
— disaggregation of dual subproblem and produced cuts ;
— implementation in a callback scheme.

We compared the basic model, the model with CPLEX built-in Benders decomposition, and Benders decomposition scheme with callbacks in two versions with different lower bound (denoted in FIG. 1 as $M1$, $M1+BD$, $BC1$ and $BC2$, respectively). Time boxplots for 10 instances with 30 activities demonstrate the efficiency of proposed scheme.

3 Conclusion

We have considered a generalized formulation of Resource Leveling Problem with variable duration of activities and continuous start/end. MILP model was enriched with Benders decomposition scheme, based on CPLEX 12.8 with callbacks and several improvements such as cuts disaggregation and lower bounds. We also compare our algorithm to the built-in CPLEX Benders decomposition. From computational tests we can conclude that for our problem proposed algorithm outperforms basic model and CPLEX built-in decomposition scheme.

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