Exact Methods for Mono-Objective and Bi-Objective Multi-Vehicle Covering Tour Problems

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1 Introduction

The *Multi-vehicle Covering Tour Problem* (MCTP) and the *Bi-Objective Multi-vehicle Covering Tour Problem* (BOMCTP) have been studied for more than thirty years. Both problems have several practical applications in industry. In this paper, we propose an effective exact method for the *Multi-vehicle Covering Tour Problem* based on column generation techniques. The effectiveness of the exact method is owed to tailored dominance rules and completion bounds. To validate our approach, we conducted extensive computational experiments on instances from literature. The comparison with state-of-the-art methods shows the effectiveness of the proposed method. In particular, seven open instances are closed to optimality for the first time, and the best lower bounds of the six open instances are improved. The exact method for the *Multi-vehicle Covering Tour Problem* is also embedded in a ϵ-constraint exact method to solve its bi-objective counterpart. Computational results show that the lower bound set provided by this bi-objective exact method is stronger than those provided by the state-of-the-art method from the literature.

2 Problem Description

The MCTP aims at finding the minimum-cost routes over a network that fulfill the requirements of different nodes. Some nodes must be visited by the vehicles whereas some other nodes have to be covered. A node is covered if it is situated within a predefined covering distance from its nearest visited node. Operational constraints on the tour lengths or the number of nodes that can visited by one vehicle are also considered. In the BOCTP, the second objective minimizes the largest distance distance between a node to be covered and a visited node.

3 Algorithm

The exact method used to solve the MCTP is based on the exact method proposed in [2] for the *Capacitated Vehicle Routing Problem*. It consists of four main steps :

1. By using column generation, compute a lower bound LB corresponding.
2. Compute an upper bound UB to the MCTP.

TAB. 1 – Computational results on the MCTP.

		Hà et al.			Notre méthode		
Instances	#	# closed	Gap	Temps_o	Time (s)	Gap	Time (s)
$\lvert V\rvert = 100$ et $\lvert M\rvert = 0$	32	**32**	17.0	37.6	**32**	**0.5**	**6.5**
$\lvert V\rvert = 100$ et $\lvert M\rvert > 0$	32	**31**	8.5	271.0	**31**	**2.4**	**5.5**
$\lvert V\rvert = 200$	32	20	13.9	1455.4	**27**	**2.0**	**393.1**
Total	96	83				**90**	

3. Generate the set $\overline{R}$ of all feasible routes with a reduced cost, with respect to the dual solution computed at Step 1, less than or equal to the gap, $UB - LB$.
4. Solve the model with the set $\overline{R}$ to optimality with a general-purpose MILP solver.

A new completion bound is proposed among other speed-up techniques. The algorithm is embedded in an ϵ-constraint method to solve the BOMCTP.

4 Computational results

The algorithm is compared to the state-of-the-art exact method for the MCTP presented in [3] and the results are reported in Table **??**. Results on the the BOCTP also reports that our algorithm outperforms the one from Artigues et al. [1].

5 Conclusions

In this paper, we proposed an exact method to solve the *Multi-vehicle Covering Tour Problem.*The efficiency of our method is proven against the state-of-the-art method for the MCTP. Some instances have now been closed thanks to our algorithm. Moreover, the single-objective method has been embedded in a well-known bi-objective technique, the ϵ-constraint method, to solve the *Bi-Objective Multi-vehicle Covering Tour Problem.*

Références

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