Social ranking rules for incomplete power relations

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1 Introduction

The problem of finding a social ranking over the set of individuals given a binary relation over the coalitions formed by them (power relation) lies in the intersection of different research domains like social choice theory, multi-criteria decision making and cooperative game theory. Examples of applications are comparing the influence of different countries in an international institution council, finding the most influential political parties in a Parliament, or rating attributes based on the role they play in multi-criteria decision situations. This problem is first studied in [1], in which authors analyze the problem from a property driven approach. Also in [2] the authors axiomatically characterize a solution based on the idea that the most influential individuals are those appearing more frequently in the highest positions in the ranking of coalitions. The authors in [3] propose a social ranking rule called Ceteris Paribus (CP-)majority rule (an adapted version of pairwise majority rule to the context of power relations) in which two individuals are ranked using information from *ceteris paribus* (i.e., everything else being equal) comparisons over all possible coalitions. To get the gist of the idea consider the power relation $1 \succ 2 \succ 3 \succ 23 \succ 13, 13 \succ 34 \succ 14 \succ 24$. [1] To compare individuals 1 and 2 CP-majority rule refers to comparisons $1 \succ 2, 23 \succ 13$, $14 \succ 24$. A plausible interpretation for these comparisons is that for example coalition 3 enables us to compare the performances of individuals $1, 2$ by examining how individuals improve performance of 3 by joining it. In this ranking rule the individual who performs better in majority of times is ranked higher (in this power relation individual 1). Another approach is suggested in [4] which ranks individuals based on their "ordinal marginal contribution". In this approach the individual whose marginal contribution is positive in most of the cooperations will be ranked higher.

As it is obvious from the above example the power relations can be incomplete and non-transitive, since in real applications forming all combinations of individuals is rarely possible. In this respect, an interesting question is whether coalitions 3 and 4 should have the same power to compare individuals. One may believe coalition 3 deserves to be more powerful in comparing individuals than coalition 4, because less comparisons made by coalition 3 may be interpreted as its expert in comparing the individuals. On the other hand, one may tend to give more power to coalition 4 because it has more experience than coalition 3 in comparing the individuals. The next question arises from the idea that considering coalitions as voters in the CP-majority rule, the size of the coalitions may differentiate their voting power: Should the coalitions of bigger size be more powerful in comparing individuals? What about the possible positive and negative synergy between individuals in a coalition? The same kind of question is relevant in the context of voting theory when the ballots of voters are not necessarily complete or transitive. In [5] the authors developed a model of aggregation (based on the idea of majority over pairs of alternatives) where the voters' preferences over a set of alternatives can be incomplete or non-transitive. They have analyzed such weighted aggregation rules

[1] Throughout the paper, we often write teams without commas and parentheses, e.g., we write 34 instead of $\{3, 4\}$.

that leverage each voter according to the size of her ballot using a property driven approach consisting of two axioms, i.e. *restricted majoritarianism* and *splitting*. They proved that for a weighted aggregation rule characterized by restricted majoritarianism the weights assigned to the voters should be constant and for those who are characterized by the splitting axiom the weight assigned to a voter should be inversely proportional to the size of her ballot. Following the same approach we investigate the impact of various factors (like the members of the coalitions, the size of the coalitions, the number of comparisons the coalitions made, etc.) in the weighted version of the CP-majority rule. More precisely suppose $\succeq_S$ represents the CP-comparisons in the power relation $\succeq$ provided by coalition S (read it the *information set* of coalition S) $\succeq_S = \{ij | \{i\} \cup S \succeq \{j\} \cup S \ s.t\ i,j \in N \setminus S\}$. We denote the general family of weighted CP-majority rules in which the weights depend on the coalitions and their information sets as $\mathcal{F}_w(\succeq) = \underset{R \in \mathcal{L}(N)}{\arg\max} \sum_{S \in 2^N} w_{\succeq_S, S} \cdot |R \cap \succeq_S|$ in which $w_{\succeq_S,S} = w(\succeq_S, S)$ is called the weight function. For simplicity of notation we refer to this family as $\mathcal{F}_{w_{\succeq_S,S}}$, which mainly indicates the structure of its weight function. We define different variations of the general family $\mathcal{F}_{w_{\succeq_S,S}}$ by restricting the domain of the weight function, which results in various sub-families of weighted CP-majority rules. As an example, reducing the domain of the weight function to the information sets of coalitions (and not coalitions) a sub-family $\mathcal{F}_{w_{\succeq_S}}$ will form as $\mathcal{F}_w(\succeq) = \underset{R \in \mathcal{L}(N)}{\arg\max} \sum_{S \in 2^N} w_{\succeq_S} \cdot |R \cap \succeq_S|$ where $w_{\succeq_S} = w(\succeq_S)$. Similarly, restricting the weight function to the sizes of the information sets generates a sub-family of $\mathcal{F}_{w_{\succeq_S}}$ as $\mathcal{F}_{w_{|\succeq_S|}}$. This hierarchical formation of weighted rules can be depicted as a tree, which has the general family of weighted CP-majority rules as the root and, going all the way down, other nodes are created by restricting the domain of the weight function with respect to the above-mentioned factors. The edges of the tree indicate the relation "being sub-family of" between any two corresponding families. Also each edge between two families corresponds to a set of axioms allowing us to characterize the sub-family of weighted CP-majority rules as members of the super-family. We are mainly interested in finding all these axioms. For instance we utilize the modified versions of splitting and restricted majoritarianism [5] to characterize respectively $\mathcal{F}_{w_{|\succeq_S|}}, w_{|\succeq_S|} = \frac{1}{|\succeq_S|}$ and $\mathcal{F}_{w_{|\succeq_S|}}, w_{|\succeq_S|} = constant$ as subfamilies of $\mathcal{F}_{w_{|\succeq_S|}}$. This work is important specially from two points. First of all defining ranking rules which are intuitively practical for different situations and needs has its out importance. Second the axiomatic study of the solutions enables us to scrutinize the implications of the ranking rules. Comparing the axioms used to characterize different sub-families from a given super-family broadens our view about how the structure of axioms change by moving from one edge to the other one. This kind of insight may provide some ideas about combining some of the axioms and forming new interesting solutions.

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