Bilevel Optimization for Collective Self-Consumption with Multiple Decision Makers

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Keywords: Collective Self-consumption, Bilevel optimization, MCCP, QCP

1 Introduction

With the fight against climate change and the need to reduce our dependence on polluting energy sources, the French government has put legislation in place to promote the adoption of distributed renewable energy production. An example of such legislation is the one on *collective self-consumption* enacted on the 26th of July 2016 [1]. A collective self-consumption agreement is one between different consumers and producers who decide to collectively consume the locally produced renewable energy.

In the proposed model, we consider a community which has multiple decision makers with conflicting objectives: an *aggregator*, who decides how the produced energy is distributed among the agreement's members, and *consumers* who minimize their energy bills in response.

This generates a hierarchical decision problem, modeled as a bilevel optimization program in which the aggregator is the leader and the agreement's members are the followers.

2 Optimization models

2.1 Bilevel model

The bilevel problem can be formulated as follows (1):

$$\min_{p \geq 0} A(p, f(p)) \tag{1a}$$

$$g_i(f_i(p), p) \leq 0 \ , \ \forall i \in \mathcal{N} \tag{1b}$$

$$f_i(p) \in \operatorname{argmin} L_i^T f_i \ , \ \forall i \in \mathcal{N} \tag{1c}$$

$$A_i f_i + D_i p \leq b_i \tag{1d}$$

The function $A(.)$ represents the aggregator's objective and $(g_i)_{i \in \mathcal{N}} \leq 0$ a set of constraints that the upper level must verify for every agent i in the set of agreement's members $\mathcal{N}$. Each agent i must minimize their objective function subject to local constraints (1d). In this case, the agent's objective function and constraints depend on both upper and lower level variables. In problem (1) the upper level variables are denoted in red and the lower level variables in blue. L_i is a vector representing the cost function of agent i. A_i and D_i are constraint matrices. b_i is a vector representing input data to the problem.

2.2 Single-level formulations

Under certain assumptions [2], we can reduce the bilevel program to a single-level by imposing the KKT conditions for lower level optimality. Based on different expressions of these optimality conditions, we obtain two equivalent single-level formulations. The first gives a Mathematical Program with Complementarity Constraints (MPCC) (1a), the second gives a Non-Convex Quadratically Constrained Program (QCP) (1b).

$$
\begin{aligned}
&\min_{p\geq 0} A(p, f)\\
&\quad \text{s.t. } \forall i \in \mathcal{N}:\\
&\quad g_i(f_i, p) \leq 0\\
&\quad A_i f_i + D_i p \leq b_i\\
&\quad A_i^T \lambda_i \leq L_i\\
&\quad \lambda_i \leq 0, f_i \geq 0\\
&\quad {\lambda_i}^T (A_i f_i + D_i p_i - b_i) = 0\\
&\quad {f_i}^T (A_i^T \lambda_i - L_i) = 0
\end{aligned}
$$

(a) First Single-level formulation: MPCC

$$
\begin{aligned}
&\min_{p\geq 0} A(p, f)\\
&\quad \text{s.t. } \forall i \in \mathcal{N}:\\
&\quad g_i(f_i, p) \leq 0\\
&\quad A_i f_i + D_i p \leq b_i\\
&\quad A_i^T \lambda_i \leq L_i\\
&\quad \lambda_i \leq 0, f_i \geq 0\\
&\quad L_i^T f_i = \lambda_i^T (b_i - D_i p_i)
\end{aligned}
$$

(b) Second Single-level formulation: QCP

The objective of this talk is to compare, through numerical experiments, the two formulations given above.

3 Numerical Results

We consider an agreement between three agents. Each one represents a residential building with energy consumption flexibilities of two different kinds: a local battery and a water heating system. We assume that the agents have invested in a collective battery, which is centrally managed by the aggregator.

By solving linear relaxations of the above single-level formulations, we obtain a lower bound for the original problem (1). By solving the lower level problem considering a fixed upper level decision, we obtain a feasible solution for the bilevel problem (1), thus an upper bound.

The table below gives the average gap $\overline{W}$ (standard deviation $\sigma(W)$) and computation time $\overline{T}$ for 30 different instances generated randomly for a collective self-consumption community of 3 buildings with simulated consumption data and realistic systems' capacities.

Results	$\overline{W}$	$\sigma(W)$	$\overline{T}$	$\sigma(T)$
MPCC	4.57×10^{-4}	8×10^{-4}	37.30 s	49.39 s
QCP	4.8×10^{-3}	5.1×10^{-3}	1.56 s	1.43 s

As shown in the above table, the MPCC formulation provides solutions with a better average gap, but a longer average computing time. For a large number of agents, the QCP formulation is more reliable.

References

[1] Code de l'énergie *Article L315-2*, CRE

[2] Stephan Dempe, Vyacheslav Kalashnikov, Gerardo A.Perez-Valdes, and Nataliya Kalashnykova, Bilevel Programming Problems: *Theory, Algorithms and Applications to Energy*, Energy systems, *Springer*, 2015.