Multi-Weighted Constrained Equal Awards in bankruptcy situation

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1 Introduction

A *bankruptcy situation* is a well-studied problem in cooperative game theory and deals with situations where an estate must be divided by n players whose demands for the estate exceed, in total, the estate. So, an allocation can not satisfy all individual demands. Several situations in practice can be analysed as bankruptcy situations. For instance, in financial situations, when a company goes bankrupt and has to pay back its shareholders or in economic applications, like how to allocate milk production quota among EU countries, or to solve environmental problems, for instance in water resources allocation problems...

Precisely, a bankruptcy situation [4] on a set of players $N = \{1, \ldots, n\}$ can be defined as a pair (E, c) where $c \in \mathbb{R}^N_+$ is a vector of player's *claims* or *demands* such that $c_i \geq 0$ for all $i \in N$, and $E \in \mathbb{R}_+$ is the estate to be shared among the players and such that $0 < E < \sum_{i=1}^{n} c_i$. As a consequence, an allocation vector $x \in \mathbb{R}^N$ is generated trying to guarantee the following properties : (i) non negativity ($x > 0$), (ii) claims boundedness ($x_i \leq c$ for any $i \in N$) and (iii) budget balance ($\sum_{i=1}^{N} x_i = E$).

Several allocation methods have been introduced in literature [5], for instance, the proportional method which divides the estate proportionally to the claims, or the constrained equal awards method [3] CEA where each claimant receives an amount equal to the smallest claim. Then, the smallest claimant drops out and the second smallest claim is fulfilled toward all claimants. Then, the second smallest claimant drops out and so on, till all players are satisfied or the estate is over or the constrained equal losses method CEL similar to CEA but the allocation starts from the biggest claimant or the Talmud a hybrid of the CEA and the CEL [2].

2 Contribution

However, players are different and each one has his own characteristics (wealth, age, health, education level...) which are ignored by classical allocation methods. In this direction, recently some authors [2], considered a richer bankruptcy situation where a second parameter is added as a *weight*. Following such approach, an allocation method does not depend only on claims, as in classical situation, but also on weights.

In this research, we focus on situations with more than one weight with the objective to take into account not only players' claims but also their multiple characteristics. Precisely, a *multi-weighted bankruptcy situation* is defined as a tuple (E, c, A) where $A \in \mathbb{R}^{N \times m}$ is a matrix specifying an m-vector of weights for each player in N.

In this paper, we introduce a new bankruptcy rule corresponding to a multi-weighted version of the constrained equal awards rule [3] [5].

Recall that for a (single-weight) weighted CEA [1], the allocation does not depend only on the smallest claim but also on the smallest weight λ, where the parameter λ is such that $\sum_{i\in N} \min\{c_i, \lambda w_i\} = E$.

Analogously, a *multi-weighted constrained equal awards* (MWCEA) allocation for a multi-weghted bankruptcy situation (E, c, A) is any allocation of E satisfying the following requirements :

$$MWCEA_i(E, C, A) = min(c_i, \lambda_1 a_{i1}, \lambda_2 a_{i2}, \ldots, \lambda_m a_{im})$$

$$with \lambda_1, \lambda_2, ...\lambda_m \in \mathbb{R}^N \text{ such that } \sum_{i\in N} min(c_i, \lambda_1 a_{i1}, \lambda_2 a_{i2}, \ldots, \lambda_m a_{im}) = E$$

for each player $i \in N$. See example in Table 1 for a 3 players game having 2 weights.

$i \in N$	c_i	$a_{1.i}$	$\lambda_{1.i}$	$a_{2.i}$	$\lambda_{2.i}$	$\lambda_1^1 = 0.25,$ $\lambda_2^1 = 0,2$	$\lambda_1^1 = 0,25,$ $\lambda_2^2 = 0,5$	$\lambda_1^1 = 0,25$ $\lambda_2^3 = 1,5$	$\lambda_1^2 = 1,$ $\lambda_2^1 = 0,2$	$\lambda_1^2 = 1,$ $\lambda_2^2 = 0,5$
1	3	3	1	2	$\frac{3}{2}$	0,4	0,75	0,75	0,4	1
2	2	1	2	4	$\frac{1}{2}$	0,25	0,25	0,25	0,8	1
3	1	4	$\frac{1}{4}$	5	$\frac{1}{5}$	1	1	1	1	1

TAB. 1 – A Generalized weighted bankruptcy situation.

Our contribution is, first, to analyse axiomatically a particular vector of $\lambda_1, \ldots, \lambda_m$ satisfying the previous conditions ; second, to apply the corresponding allocation on a real case : the allocation problem of CO2 emission permits. In fact, natural resources allocations problems have been already studied as a bankruptcy situation. In previous studies [6], the allocation of CO2 emissions permits was generated by WCEA rule taking into account countries' claims and countries' Gross Domestic Product (GDP), as parameters. However, CO2 emissions permits is a sensitive problem and it is obvious to consider more criteria, like population, or energy, or emission intensity. That is why, in this paper, we compare the results of the multi weighted CEA rule with those of the WCEA and the CEA rule, to allocate CO2 emission permits for the EU-28. To do so, we define countries' emissions as claims, the GDP as the first weight for both WCEA and MWCEA and Population as a second weight for the MWCEA. As results of the MWCEA allocations, some countries are totally satisfied like Croatia, Portugal, Sweeden, Romania, Hungary and France. Some of them were disadvantaged with WCEA and CEA. On the other side, other countries, like UK and Germany, get less than their WCEA allocations but the new allocations are better than CEA allocations. We finally observe that the CEA allocation favors countries with low demands, WCEA favors productive countries and MWCEA keeps into account a richer description of the problem.

Références

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