The notion of crossing task for the cumulative scheduling problem and how to use it to compute lower bounds of the optimal makespan

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Mots-clés : *scheduling, makespan, lower bound, crossing task*

1 Abstract

In the Cumulative Scheduling Problem (CuSP) [2, 3, 4, 5], the aim is to schedule without preemption a set I of n independent taks on a single resource composed of m identical machines. With each task i are associated a release date r_i, a processing time p_i, a tail q_i, and a resource consumption c_i, *i.e.* task i requires simultaneously c_i machines during its processing. The aim is to find a schedule minimizing the makespan C_{max}. Denoting by C_i the completion time of task i in such a schedule, we have : $C_{max} = \max_{i \in I}\{C_i + q_i\}$. We will present our results for the m machines problem for which all the c_i are equal to 1 and we generalize them on the CuSP.

Being given a trial value of C_{max}, a deadline $d_i = C_{max} - q_i$ can be associated with each task i. The question is then to know if there exists a feasible schedule satisfaying the deadlines.

Définition 1 *Given a makespan C_{max}, a task i is called a* punctual crossing task *if and only if there exists an interval of time in which task i is necessarily scheduled, i.e. if there exists t such that $d_i - p_i \leq t < r_i + p_i$. If task i is necessarily scheduled during interval $[t-1, t]$, we say that i is a t-punctual crossing task ($t \in \{1, \ldots, C_{max}\}$).*

Proposition 1 *Given a makespan C_{max}, for each punctual crossing task i we have $r_i + 2p_i + q_i > C_{max}$.*

From this proposition, we can deduce that if $\max_{i \in I}\{r_i + 2p_i + q_i\} \leq C_{max}$, then there is no punctual crossing task. In the sequel, we denote by C^*_{max} the makespan associated with an optimal non-preemptive schedule.

An immediate consequence is that if there are strictly more than m t-punctual crossing tasks, then no non-preemptive schedule with a makespan less than or equal to C^*_{max} exists.

This result allows an alternative lower bound LB_1 for the CuSP associated with a non-preemptive schedule to be proposed. It can be computed in $O(n \log n)$. The approach aims to adjust C_{max} by computing the actual punctual crossing tasks and performing its increasing whenever this number exceeds m.

The following theorem can be stated :

Theorem 1.1 *If $C^*_{max} > \max_{i \in I}\{r_i + 2p_i + q_i\}$ then I contains at least $2m$ tasks and there is no punctual crossing task. Moreover, the quantity*

$$LB_2 = \min_{\substack{K=\{k_1,\ldots,k_m\}\subset I,\\ J=\{j_1,\ldots,j_m\}\subset I,\\ K\cap J=\emptyset}} \left\{ \frac{1}{m}(r_{k_1} + \ldots r_{k_m}) + \frac{1}{m}\sum_{i\in I} p_i + \frac{1}{m}(q_{j_1} + \ldots q_{j_m}) \right\}$$

is a lower bound of the makespan.

Next we come back to the general case when there is no assumption on C^*_{max} and we generalize the previous theorem. Finally we show its connection with energetic reasoning [1, 6].

Acknowledgment : This work partially financed by the project 2018-0062H of the *Gaspard Monge Program for Optimization, operations research, and their interactions with data science.*

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