Column generation decomposition for Variable Delay Multi-Commodity Flow problem

Nicolas HUIN[1] Jérémie Leguay[1] Sébastien Martin[1]

Huawei Technologies prénom.nom@huawei.com

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1 Introduction

5G promises to improve network performance in the way of delay, bandwidth or scalability: an increasing number of services can be moved to the cloud and the edge, putting the computation burden of services such as video game streaming, V2X or AI assistant away from the customers. But these types of services requires stricter Quality of Service (QoS) constraints (e.g., delay, losses, jitter) and satisfying them is crucial.

Modeling the delay from the node queuing system is a well studied problem but only few works have tackled it from as a global optimization problem. Fortz *et al* considered the piecewise linear unsplittable multicommodity flow problem [1]: we generalize their model by considering other delay models.

In this paper, we focus on using more realistic model of delay and propose a new decomposition based on column generation for solving the multi-commodity flow problem when the delay is not constant (called Variable Delay Multi-commodity flow problem). We propose a decomposition model leveraging the traffic on the nodes.

2 Problem statement and model

Let $G = (V, E)$ be a graph representing the network. Each link $e \in E$ is characterized by its cost (C_e), its propagation delay (λ_e) and its bandwidth capacity (B_e). Let K be the set of requests to provision on the network. Each demand $k \in K$ is characterized by its source (s_k), its destination (t_k), its bandwidth requirement (D_k) and its delay requirement (Λ_k).

The goal of the Variable Delay Multi-Commodity Flow (VD-MCF) problem is to provision all requests in the network, respecting the link capacity constraints and requests delay constraints, while minimizing the cost of the network usage.

For each link $e \in E$ and each demand $k \in K$ we consider a binary variable y_e^k equals to 1 if the demand k uses the link $e \in E$ and 0 otherwise. We also consider for each node $v \in V$ and for each subset $K' \subseteq K$ a binary variable $x_v^{K'}$ equals to 1 if the set of demand K' traverses the node v and 0 otherwise. $\delta_v^{K'}$ corresponds to the delay on v when its processing the demands in K'.

The following integer linear program solves the Variable Delay Multi-commodity flow problem.

$$\min \sum_{e \in E} C_e \sum_{k \in K} D_k y_e^k$$

$$\begin{aligned}
\alpha_{ud}: &\quad \sum_{e \in \delta^+(v)} y_e^k - \sum_{e \in \delta^-(v)} y_e^k = \begin{cases} 1 \text{ if } v = s_k \\ -1 \text{ if } v = t_k \\ 0 \text{ otherwise} \end{cases} && \forall v \in V, \forall k \in K \\
\alpha_e: &\quad \sum_{k \in K} D_k y_e^k \leq B_e && \forall e \in E, \\
\beta_{ud}^1: &\quad y_{uv}^k \leq \sum_{K' \subseteq K : k \in K'} x_u^{K'} && \forall uv \in E, \forall k \in K, \\
\beta_{vd}^2: &\quad y_{uv}^k \leq \sum_{K' \subseteq K : k \in K'} x_v^{K'} && \forall uv \in E, \forall k \in K, \\
\beta_v: &\quad \sum_{K' \subset K} x_u^{K'} \leq 1 && \forall v \in V, \\
\beta_k: &\quad \sum_{u \in V} \sum_{K' \subseteq K : k \in K'} \lambda_u^{K'} x_u^{K'} + \sum_{e \in E} \lambda_e y_e^k \leq \Lambda_k && \forall k \in K,
\end{aligned}$$

Inequalities α_v are the flow constraints. Inequalities α_e are the capacity constraints. Inequalities β_{vk}^1 and β_{vk}^2 ensure that if a link uv is used by a demand k then at least one subset K' containing k is activated on u and v. Inequalities β_v guarantee that only one subset of demands is activated for each node. Inequalities β_k are delay constraints.

Variables $x_v^{K'}$ are in exponential number thus it is necessary to generate them using column generation procedure. Let $\bar{x}_k$ be a binary variable equal to 1 if the demand k is selected, the objective function of the pricing problem is to maximize $\sum_{k \in K} (\beta_{uk}^1 + \beta_{uk}^2) \bar{x}_k - y_{K'}$ where $y_{K'}$ depends on the delay model used such as:

- **Constant value**, $y_{K'} = \sum_{k \in K} \beta_k \bar{x}_k$,
- **Linear function**, $y_{K'} = a_v \sum_{k \in K} \beta_k D_k \bar{x}_k$, where a_v correspond to an arbitrary linear delay coefficient,
- **Queue model**, $y_{K'} = \frac{\sum_{k \in K} \beta_k \bar{x}_k}{\sum_{e \in \delta^+(v)} \left(B_e - \sum_{k \in K} D_k \bar{x}_k\right)}$

3 Conclusion

We present a new decomposition model for solving the Variable Delay Multi-commodity Flow (VD-MCF) problem. The strength of the model comes from the fact that we can plug any kind of delay model in the pricing problem: from a simple M/M/1 queuing system to more complex system, where the delay evaluation could be done using machine learning algorithm.

References

[1] Bernard Fortz, Luís Gouveia, and Martim Joyce-Moniz. Models for the piecewise linear unsplittable multicommodity flow problems. *European Journal of Operational Research*, 261(1):30 – 42, 2017.