Strong RLT1 bounds for some quadratic 0–1 optimization problems with linear constraints computed from decomposable Lagrangean relaxations

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decomposable Lagrangean relaxation.

The purpose of this research is to generate quickly strong bounds for pure (0-1) quadratic problems with linear constraints. The basis of our approach is the classical Reformulation Linearization Technique (RLT) of Sherali and Adams (1986, 1990). When applied to a pure 0-1 quadratic optimization problem with linear constraints (P), RLT constructs a hierarchy of LP (i.e., continuous and linear) models of increasing sizes, which provide monotonically improving continuous bounds on the optimal value of (P) as the level, i.e., the stage in the process, increases. When the level reaches the dimension of the original solution space, the last model provides an LP bound equal to the IP optimum. In practice, unfortunately, the problem size increases so rapidly that for large instances, even computing bounds for RLT models of level k (called RLTk) for small k may be challenging. To our knowledge, only results for bounds of levels 1, 2, and 3 have been reported in the literature.

We are proposing, for certain quadratic problem types, a way of producing stronger bounds than continuous RLT1 bounds in a fraction of the time it would take to compute continuous RLT2 bounds. The approach consists in applying a specific decomposable Lagrangean relaxation to a specially constructed RLT1-type linear 0–1 model. If the overall Lagrangean problem does not have the integrality property, and if it can be solved easily as a 0–1 rather than a continuous problem, one obtains 0–1 RLT1 bounds of roughly the same quality as standard continuous RLT2 bounds, but in a fraction of the time and with much smaller storage requirements. In (Guignard 2018), the quality of the bounds was ascertained, but the decomposition was only partially implemented. We will show that the same bounds, obtained by fully decomposing the Lagrangean models, can be obtained in still much smaller computer times.

We will present numerical results for the Cross-dock Door Assignment Problem, a special case of the Generalized Quadratic Assignment Problem, and for the 0–1 Quadratic Knapsack Problem. These show that just solving one decomposable Lagrangean relaxation problem in 0–1 variables produces a bound close to or better than the standard continuous RLT2 bound (when available) but much faster, especially for larger instances.

References

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