An ACOPF formulation primer

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1 Introduction

The Alternating Current Optimal Power Flow (ACOPF) problem is one of the most important problems arising in the energy industry [1]. The basic physical quantity in electromagnetism is the charge. The electromagnetic field is the force vector acting on a unit charge at each point of a the three-dimensional domain. The current I measures the rate of charge traversal of a surface unit per second. The voltage V is the potential energy of a unit charge in the electromagnetic field. The power S is the product of current and voltage. The power induced on a line by the voltage difference at the line endpoints varies with time because of the alternating nature of the power generation process. Since studying power as a periodic function of time would be impossibly CPU-intentive, averages of the power power function over time periods are considered instead. The ACOPF is usually cast as a Mathematical Programming (MP) problem over the complex numbers: the imaginary part provides an average over time of an orthogonal power function.

The ACOPF asks for the best power flow over an electrical network modelled by a digraph $N = (B, L)$, where B is the set of *buses* (nodes) and L the set of *lines* (arcs). We also introduce a set G of generators installed at buses: potentially, any number of generators may be installed at a given bus. The standard ACOPF can be reformulated as a (larger) MP over the reals by separating real and complex parts [4]. The ACOPF is **NP**-hard [2]. While objective functions vary in the literature, it is common to consider linear or quadratic objectives with respect to power. While the ACOPF only has continuous variables, more realistic variants include binary variables which activate/deactivate various electrical components [3]. The activation/deactivation of generators defines an ACOPF variant called ACOPF WITH GENERATORS (ACOPFG) [5]

The ACOPF poses formidable modelling difficulties, partly because of the complex number setting, and partly because of the data representation chosen by electrical engineers. The open-source academic software reference, which provides a data format, modelling platform, and solvers, is MATPOWER [6]. MATPOWER has numerous qualities, but it does not come across as "easy to use" by OR researchers and practitioners. In this talk we discuss some of the ACOPF modelling difficulties and how to address them.

2 The asymmetric nature of transformer-endowed lines

We recall some complex number notation: any $x \in \mathbb{C}$ can be represented by separating real and imaginary parts as $x = x^{\mathsf{r}} + ix^{\mathsf{c}}$, where $i = \sqrt{-1}$. Its *complex conjugate* is $x^* = x^{\mathsf{r}} - ix^{\mathsf{c}}$; its *modulus* is $|x| = \sqrt{xx^*}$; its *phase* is $\vartheta_x = \arccos(x^{\mathsf{r}}/|x|) = \arcsin(x^{\mathsf{c}}/|x|)$.

The foremost difficulty OR-educated people meet when modelling the ACOPF is possibly the presentation of the data related to the network N. A line between buses b and a is an abstraction of an electrical cable. Since AC electricity has a frequency of 50-60Hz, power is induced on the line at b but also at a, depending on the oscillations of voltage at b, a. Thus the line is represented by pairs of antiparallel arcs (b, a) and (a, b) (there may also be parallel lines, representing parallel cables, which we do not deal with here). The main issue of modelling the ACOPF stems from the fact that, unlike with DC, the generalization of Ohm's law yields different current magnitudes on the line according to the direction. We define decision variable vectors $\mathbf{I}_{ba} = (I_{ba}, I_{ab}) \in \mathbb{C}^2$ for current on the line, and $\mathbf{V}_{ba} = (V_b, V_a) \in \mathbb{C}^2$ for voltage at the line endpoints, as well as a constant 2×2 complex matrix $\mathbf{Y}_{ba}$ [1, Fig. B.1]. Then Ohm's law for the line between b and a is $\mathbf{I}_{ba} = \mathbf{Y}_{ba}\mathbf{V}_{ba}$.

Decision variables $S_{ba} \in \mathbb{C}$ denote the power injected on the line at b, and are subject to $S_{ba} = V_b I_{ba}{}^*$. Other decision variables $\mathscr{S}_g$, for g being a generator in $\mathscr{G}_b$, which is the set of generators assigned to bus B, denote the power generated at g. The main equations regulating the power flow are Kirchhoff's laws:

$$\forall b \in B \qquad \sum_{(b,a)\in L} S_{ba} + \tilde{S}_b = -A_b{}^*|V_b|^2 + \sum_{g\in\mathscr{G}_b} \mathscr{S}_g, \tag{1}$$

where A_b is a constant related to interaction with the ground, and $\tilde{S}_b$ is the power demand at bus b. The formulation is completed by bound constraints: on generated power $\mathscr{S}$, on injected power magnitude $|S_{ba}|$, and on voltage magnitude $|V_b|$ (bounds on voltage phase difference, not treated here, may also be enforced).

Usually, S_{ba} is replaced by $V_b I_{ba}{}^*$, and $\mathbf{I}_{ba}$ by $\mathbf{Y}_{ba}\mathbf{V}_{ba}$, which gives a voltage-only formulation. This replacement, however, also yields an indexing problem: it lets us obtain injected powers in antiparallel pairs S_{ba}, S_{ab}, but S appears in Eq. (1) in cutsets around $\{b\}$, making the replacement technically challenging. This can be addressed by rewriting Eq. (1) as follows:

$$\forall b \in B \qquad \sum_{(b,a)\in L_0} S_{ba} + \sum_{(b,a)\in L_1} S_{ba} + \tilde{S}_b = -A_b{}^*|V_b|^2 + \sum_{g\in\mathscr{G}_b} \mathscr{S}_g, \tag{2}$$

where $L_0 \cup L_1 = L$ and $(b, a) \in L_0 \leftrightarrow (a, b) \in L_1$. We then assume that all of the lines between b, a with transformers at b are in L_0. Then $S_{ba} = V_b I_{ab}{}^*$ for each $(b, a) \in L_1$.

By separating real and imaginary parts, and assuming a linear objective in generated power, a complex ACOPF formulation can be reduced to a voltage-only Quadratically Constrained Quadratic Program, which can be locally solved using some local Nonlinear Programming solvers (satisfactory results were obtained using Baron limited to preprocessing).

References

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