

# Primal-dual approach to the multi-activity tour scheduling problem

Stefania Pan<sup>1</sup>, Mahuna Akplogan<sup>4</sup>, Lucas Létocart<sup>2</sup>,  
Louis-Martin Rousseau<sup>3</sup>, Nora Touati<sup>4</sup>, Roberto Wolfler Calvo<sup>2</sup>

<sup>1</sup> Artelys, 81 Rue Saint-Lazare, 75009 Paris, France  
`stefania.pan@artelys.com`

<sup>2</sup> LIPN, UMR 7030 CNRS, Université Paris 13,  
99 avenue Jean-Baptiste Clément 93430, Villetaneuse, France  
`letocart,wolfler@lipn.fr`

<sup>3</sup> CIRRELT - Polytechnique Montréal, CP6079 Succ Centre Ville,  
Montréal, Canada, H3C3A8,

<sup>4</sup> Horizontal Software, 9 Rue de l'Isly, 75008 Paris, France  
`ntouati@horizontalsoftware.com`

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## 1 Introduction

Personnel scheduling problems consists of constructing feasible shift schedules to be assigned to staff, in order to satisfy workload requirements. These problems arise in several organizations such as airline and railways companies, hospitals, restaurants, retail stores and call centres. Due to economic considerations, personnel scheduling represents an intense and challenging research field [1]. Three main categories of problems can be distinguished in personnel scheduling : shift scheduling, days-off scheduling and tour scheduling. In this work, we deal with a problem in the latter category. We aim at specifying the time periods of the day and the days of the week in which employees have to work. Moreover, more than one work activity has to be scheduled, making the problem fall into multi-activity tour scheduling category. In these problems, we need not only to define the working days and the working periods, but also to specify the allocation of work activities.

## 2 Primal-dual approach

We solve the multi-activity tour scheduling problem by combining column generation and large neighborhood search into a primal-dual approach. The first solves the linear relaxation providing lower bound and fractional solutions, while the second starts from integer rounded solutions and aims at improving them, providing upper bounds on the integer problem.

**Column generation (CG).** CG is a classical technique to solve linear programs with a large number of variables. Recently, CG approaches have been widely used to solve multi-activity tour scheduling problems [1]. This method considers on one hand a master problem that takes into account workload requirements, minimizing the total cost given by under and over coverage. On the other hand, new feasible schedules are built solving the subproblems, where the legal constraints, such as consecutive working hours, daily working hours, breaks and skills, are considered. Each subproblem is decomposed and solved in different phases. First (*phase 1*), work activities are combined to build feasible timeslots, which are in turn used to build feasible daily shifts (*phase 2*). Finally (*phase 3*), daily shifts are combined to build

feasible weekly schedules. Rules defining timeslots and daily shifts are taken into account both by means of automata, and by solving resource constrained shortest path problems on the extended graphs.

**Large Neighborhood Search (LNS).** The LNS algorithm was first introduced by [3], and it iteratively destroys part of the current solution and repairs it in the hope of finding a better solution. Similarly to [2], destroying here means choosing an employee and removing his schedule, while repairing means assigning a new schedule to the selected employee in order to improve the global solution.

**The overall primal-dual approach.** The proposed primal-dual approach iteratively calls CG and LNS at each iteration. It starts by solving the linear relaxation using CG. When optimality is achieved, a lower bound and a fractional solution are available. The latter is rounded to provide a feasible integer solution which is improved by means of LNS. As soon as a local optimum is found, the best solution is given as initial solution to CG, which again solves the linear relaxation and stops when the value of the master problem is close to optimality. The overall primal-dual heuristic stops when the gap between lower and upper bounds is less than 1%, or a maximum number of iterations is exceeded.

Preliminary computational results on instances built with input from quick service restaurants, show that the primal-dual method proposed allows an average gap of 5% between the lower bound given by the linear relaxation and the upper bound of the best integer solution obtained.

### 3 Conclusions et perspectives

The primal-dual approach proposed makes use of CG to get lower bounds and fractional solutions, and LNS to get upper bounds and feasible integer solutions. Computational results show that the method finds good quality solutions for all the tested instances. We aim to embed the primal-dual approach in a branch-and-price framework to find good feasible integer solutions during the search tree.

### Références

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