

Stable coalition structures based on social rankings.

Is it better to be a big fish in a small pond or a small fish in a big pond ?

Stefano Moretti¹

LAMSADE, CNRS, Université Paris-Dauphine, Université PSL,
Place du Maréchal de Lattre de Tassigny, F-75775 Paris cedex 16, France
`stefano.moretti@dauphine.fr`

Mots-clés : *social ranking, hedonic games, ordinal power indices, coalition structures*

1 Introduction

A *social ranking solution*, or briefly a *social ranking*, over a set $N = \{1, \dots, n\}$ is defined as a map assigning to each *power relation* (i.e. a ranking over subsets of N) another ranking over the single elements in N . A first notion of social ranking has been recently introduced in [9] as the ranking provided by a classical solution concept for cooperative games, the *Banzhaf value*[3], applied to games whose Banzhaf ranking is invariant to the choice of the characteristic function. Differently, in [10], a property-driven approach is used to obtain several impossibility results proving that no *social ranking* satisfies a given set of attractive properties. A social ranking solution based on a majoritarian principle has been studied in [7], where two individuals are compared on the basis of their respective performance when added to all the other possible coalitions. A social ranking based on the idea that the most influential individuals are those appearing more frequently in the highest positions in a power relation has been introduced and axiomatically characterized in [4]. Other social ranking solutions based on the idea of ordinal marginal contribution have been recently introduced in [8]. The manipulability of social rankings from the literature has also been studied in [1], focusing on situations where individuals may be interested in weakening their group's effectiveness in order to reach a better position in the corresponding social ranking.

Another important class of cooperative interaction situations is the one of *hedonic games* [6], where players of a set N have preferences over coalitions they can form. The analysis of hedonic games mainly focus on partitions of the set N into disjoint coalitions, which are in general referred to as *coalition structures*. A coalition structure may be *stable* according to various notions of stability (see, for instance, [2, 5]). In particular, we will consider the notion of *core stability* : a coalition structure is core stable if there is no coalition $S \subseteq N$ such that all players in S strictly prefer to form S than staying in their respective coalitions within the coalition structure.

In this paper we consider a particular class of hedonic games where the preferences of the agents over coalitions are induced by alternative notions of social ranking. More precisely, agents compare coalition structure keeping into account both the ranking of coalitions to which they belong (according to the power relation) and the social ranking they can obtain in those coalitions (here a social ranking over a coalition S is computed considering the restriction of the power relation to coalition S).

As an example, consider a situation with $N = \{1, 2, 3\}$ and a power relation over non-empty coalitions such that

$$\{1, 2, 3\} \succ \{1, 2\} \succ \{2, 3\} \succ \{1, 3\} \succ \{1\} \succ \{2\} \succ \{3\},$$

where $S \succ T$ means that “coalition S is strictly stronger than coalition T ”, for any $S, T \subseteq N$. Suppose that the grand coalition N forms (remember that we are looking for stable coalition

structures ; so, if the most powerful coalition N forms, the corresponding coalition structure is $\{N\}$ and that, within N , individual 1 is the leader with the highest social ranking, whereas 3 has the lowest social ranking. Suppose also that if coalition $\{2, 3\}$ forms then 2 is the leader with the highest social ranking. Since, coalition $\{2, 3\}$ is strictly stronger than $\{1\}$, players 2 and 3 are both willing to exclude player 1 from cooperation : both 2 and 3 remain in the most powerful coalition of the new coalition structure $\{\{2, 3\}, \{1\}\}$ and, in addition, they strictly increase their respective social ranking. On the other hand, if for some reasons the social ranking for coalition $\{2, 3\}$ was such that 3 was the leader within $\{2, 3\}$, then player 2 would not get any improvement of social ranking from breaking the cooperation with player 1.

Based on this kind of preferences for players over coalition structure, we show that in general we cannot guarantee the existence of stable coalition structures, except for some families of particular power relations and using particular social rankings among those introduced above. For instance, if we restrict the domain of power relations to homogeneous ones (i.e., larger coalitions are more powerful) and the social ranking is “consistent” over all coalitions (it preserves the ordering of the players) then we cannot guarantee the existence of a stable coalition structure.

Références

- [1] Allouche, T., Escoffier, B., Moretti, S., Öztürk, M. (2019) Social ranking manipulability for the CP-Majority, Banzhaf and Lexicographic Excellence solutions. (submitted)
- [2] Aziz, H., Brandt, F., Seedig, H.G. (2013) Computing desirable partitions in additively separable hedonic games. *Artificial Intelligence*, vol. 195, pp. 316-334.
- [3] Banzhaf III, J.F. (1964) Weighted voting doesn't work : A mathematical analysis. *Rutgers Law Review*, vol. 19, p. 317.
- [4] Bernardi, G., Lucchetti, R., Moretti, S. (2019) Ranking objects from a preference relation over their subsets. *Social Choice and Welfare*, pp. :1-18.
- [5] Ciftci, B.B., Dimitrov, D.A. (2006) Stable Coalition Structures in Simple Games with Veto Control. CentER Discussion Paper, 2006.
- [6] Dreze, J.H., Greenberg, J. (1980) Hedonic coalitions : Optimality and stability. *Econometrica* (pre-1986), vol. 48, no. 4, p. 987, 1980.
- [7] Haret, A., Khani, H., Moretti, S., Öztürk, M. (2018) *Ceteris Paribus* Majority for social ranking. Proceedings of the 27th International Joint Conference on Artificial Intelligence (IJCAI-18), pp. 303-309.
- [8] Khani, H., Moretti, S., Öztürk, M. (2019) An Ordinal Banzhaf Index for Social Ranking. Proceedings of the 28th International Joint Conference on Artificial Intelligence, pp. 378-384.
- [9] Moretti, S. (2015) An axiomatic approach to social ranking under coalitional power relations. *Homo Oeconomicus*, 32(2) : 183-208.
- [10] Moretti, S., Öztürk, M. (2017) Some Axiomatic and Algorithmic Perspectives on the Social Ranking Problem. In : Proceedings of the fifth International Conference on Algorithmic Decision Theory (ADT 2017) Luxembourg, Springer.
- [11] Razin, R., Piccione, M. (2009) Coalition formation under power relations. *Theoret Economics*, vol. 4(1), 1-15.