An enhanced multicut stochastic Benders decomposition algorithm for network design problem

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Key-words : *Stochastic Optimization, Benders Decomposition*

1 Context

RTE (Réseau de Transport d'Electricité) is the French public company in charge of the power system management. Based on French legislation, they have to produce every year a forecast report. This report aims to identify the risks of imbalance between the production and the electricity demand to horizons of 5 to 15 years. For this analysis, a stochastic optimization problem is solved. This problem models the costs of investments to build or to maintain production plants and transport infrastructures. It also takes into account the variable costs of production and transportation needed to satisfy the balance between supply and demand according to different scenarios. This problem is solved with Benders decomposition and the resolutions are currently limited in the number of scenarios, because of the size and the number of problems involved.

2 Litterature

Benders decomposition is a broadly-used method to solve two-stage stochastic programs. As it is known to be slow to converge, a large amount of research has been done to find enhancement methods. As far as we know, an important part of the research deals with primal or dual stabilizations of Benders decomposition [1, 2]. In our case, one of the major bottlenecks faced is the large number of subproblems that we have to solve at each iteration. In the case of stochastic optimization, the stochasticity has been managed either by massive parallelisation, or by statistical methods in a classical monocut framework [3, 4]. In the latter case, either the validity of the cuts was not preserved, or the solutions were only statistically viable.

3 Contribution

As stochastic gradient descent algorithm allowed machine learning researchers to solve larger and larger stochastic one-stage optimization problems, we transpose the idea to solve only a few realizations of the random variable at each iteration to Benders decomposition. Our contribution supposes that we implement a multicut decomposition [5]. We present an exact method, which allows to scale-up the number of scenarios we can model in a network design problem. We first show that the sampling of only few realizations does not affect the exactness of the method. As the subproblems are not all evaluated, we do not have anymore an upper bound on the objective function, we then propose an adapted stopping criterion. Finally, we analyze multiple schemes to sample scenarios. Numerical experiments are performed, both on actual RTE data and on randomly generated data. We discuss the effectiveness of the method compared to a stabilized multicut Benders implementation. The method seems to be structurally adapted to stochastic programs. The first results are really promising and some works will be done to apply it to other types of problems.

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