Near-Optimal Robust Bilevel Optimization

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1 Introduction

Bilevel optimization studies problems where the optimal response to a second mathematical optimization problem is integrated in the constraints. Such structure arises in a variety of decision-making problems in areas such as market equilibria, policy design or product pricing.

We introduce the concept of near-optimal robustness for bilevel problems, protecting the upper-level decision-maker from bounded rationality at the lower level and show it is a restriction of the corresponding pessimistic bilevel problem. Essential properties are derived in generic and specific settings. This model finds a corresponding and intuitive interpretation in various situations cast as bilevel optimization problems. We develop a duality-based solution method for cases where the lower level is convex, leveraging the methodology from robust and bilevel literature. The models obtained are tested numerically using different solvers and formulations, showing the successful implementation of the near-optimal robust bilevel problem.

Solving bilevel problems under limited deviations of the lower-level variables was introduced in [1] under the term "ε-approximation" of the pessimistic bilevel problem. The authors define special properties and a solution method for this variant in the so-called independent case, *i.e.* where the lower-level feasible set is independent of the upper-level decision. We generalize the approach of [1] to problems involving upper- and lower-level variables in the constraints at both levels.

2 Bilevel optimization and proposed model

In this paper, we consider the near-optimal robust versions of bilevel problems *NORBiP*, which we define as :

$$\min_{x,v} \; F(x,v) \tag{1a}$$

s.t.

$$G_k(x,v) \leq 0 \qquad \forall k \in [\![m_u]\!] \tag{1b}$$

$$x \in \mathcal{X} \tag{1c}$$

$$v \in arg\min_{y}\{f(x,v) \; s.t. \; g(x,v) \leq 0, y \in \mathcal{Y}\} \tag{1d}$$

$$G_k(x,z) \leq 0 \qquad \forall k \in [\![m_u]\!], \forall z \in \mathcal{Z}(x;\delta) \tag{1e}$$

$$\text{with } \mathcal{Z}(x;\delta) = \{y \mid g(x,y) \leq 0, y \in \mathcal{Y},\; f(x,y) \leq f(x,v) + \delta\} \tag{1f}$$

where n_l, n_u are the number of lower- and upper-level variables respectively, m_l, m_u are the number of lower- and upper-level constraints respectively, $\mathcal{X} \subseteq \mathbb{R}^{n_u}$, $\mathcal{Y} \subseteq \mathbb{R}^{n_l}$. We use the notation $[\![a]\!] \equiv \{1, 2..a\}$ for any natural number $a > 0$. The upper- and lower-level objective functions are noted $F, f : \mathcal{X} \times \mathcal{Y} \mapsto \mathbb{R}$ respectively. Constraint (1b) and $g(x, y) \leq 0$ are the upper- and lower-level constraints respectively.

3 Interpretation as Stackelberg games

In the context of bilevel problems modeling Stackelberg games, when optimizing their objective function, the leader (upper-level problem) anticipates an optimal reaction of the follower (lower-level problem) to their decisions. However, in many practical cases, the follower makes near-optimal decisions [1], meaning decisions resulting in a limited deviation of the lower-level objective. An important issue in this setting is the definition of the robustness of the leader's decisions with respect to near-optimal followers ones.

An interpretation for a near-optimal decision of the follower has been developed in game theory as *bounded rationality*. The concept was initially proposed in [2], and is sometimes referred to as ε-rationality [3]. Bounded rationality defines an economic and behavioral interpretation of a decision-making process where an agent aims to take *any* solution associated with a "satisfactory" objective value instead of the optimal value.

4 Solution concept

Solution concepts are designed for models where the lower-level problem is convex and linear. In such cases, duality theory can be used to derive tight bounds on the worst-case of the near-optimal set. Furthermore, KKT conditions are sufficient to characterize the optimality of a lower-level solution. These two components let us reformulate the near-optimal robust problems in single-level, closed-form equivalents. An algorithm is designed from the reformulation, eliminating rapidly infeasibility cases. Computational results highlights its efficiency to solve randomly generated linear-linear bilevel problems with near-optimality robustness constraints.

Références

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