# Algorithmic Multistage Optimization

Evripidis Bampis[1], Bruno Escoffier[1], Alexandre Teiller[1]

Sorbonne Université, CNRS, LIP6, F-75005 Paris, France, and
Institut Universitaire de France
{evripidis.bampis,bruno.escoffier,alexandre.teiller}@lip6.fr

**Keywords** : Multistage optimization ; Online algorithms ; Approximation

## 1 Introduction

Many systems have to be maintained while the underlying constraints, costs and/or profits change over time. Since in many applications changing the solution is costly, the task becomes to find a sequence of solutions that optimizes the trade-off between good per-time solutions and stable solutions taking into account an additional similarity bonus/transition cost. In order to model such situations, Gupta et al. [4] and Eisenstat et al. [3] recently proposed a *multistage* model where given a time horizon $t = 1, 2, \ldots, T$, the input is a sequence of instances $I_1, I_2, \ldots, I_T$, (one for each time step), and the goal is to find a sequence of solutions $S_1, S_2, \ldots, S_T$ (one for each time step) reaching a trade-off between the quality of the solutions in each time step and the stability/similarity of the solutions in consecutive time steps.

## 2 Offline setting

The addition of the transition cost makes some classic combinatorial optimization problems much harder. This is the case for instance for the minimum weighted perfect matching problem in the *offline* case where the whole sequence of instances is known in advance. While the one-step problem is polynomially-time solvable, the multistage problem becomes hard to approximate even for bipartite graphs and for only two time steps [1, 4]. In the *offline* setting, another natural problem to examine is the Knapsack problem. We propose a PTAS for the Multistage Knapsack problem. This is the first approximation scheme for a combinatorial optimization problem in the considered multistage setting, and its existence contrasts with the inapproximability results for other combinatorial optimization problems that are even polynomial-time solvable in the static case. Then, we prove that there is no FPTAS for the problem even in the case where $T = 2$, unless $P = NP$. Furthermore, we give a pseudopolynomial time algorithm for the case where the number of steps is bounded by a fixed constant and we show that otherwise the problem remains NP-hard even in the case where all the weights, profits and capacities are 0 or 1.

## 3 Online and lookahead setting

In the *online* case, at time $t$ no knowledge is available for instances at times $t + 1, \ldots, T$. In the *k-lookahead* case, at time $t$ the instances at times $t + 1, \ldots, t + k$ are also known. Our goal is to measure the impact of the lack of knowledge of the future on the quality and the stability of the returned solutions. Indeed, our algorithms are limited in their knowledge of the sequence of instances. Given that the number of time steps is given, we compute the competitive ratio of the algorithm after time step $T$.

Some recent results are already known for the online multistage model [2, 4], however all these results are obtained for specific problems. We show that a wide variety of problems shares some properties in the multistage framework, especially the set of *subset maximization problems* which represent numerous combinatorial optimization problems (knapsack, maximum-weight matching, etc.) and can be expressed as follows : One is given a ground set $N = \{1, \ldots, n\}$, a collection $\mathcal{F} \subseteq 2^N$ of subsets thereof such that $\emptyset \in \mathcal{F}$, and an objective (profit) function $p : \mathcal{F} \rightarrow \mathbb{R}_+$. The task is to choose a set $S \in \mathcal{F}$ that maximizes $p(S)$. The profit function $p_t$ (and possibly the set of feasible solutions $\mathcal{F}_t$) may change over time. As transitioning from one state (solution) to another often introduces a non-negligible cost in these problems, we introduce two similarity measures : the *Intersection Bonus* where the bonus is proportional to the number of objects in the solution at time $t$ that remain in it at time $t+1$ and the *Hamming Bonus* where we get the bonus for each object for which the decision (to be in the solution or not) is the same between time steps $t$ and $t+1$.

We develop general techniques for online multistage *subset maximization problems* and thereby characterize those models (given by the type of data evolution and the type of similarity measure) that admit a constant-competitive online algorithm. When no constant competitive ratio is possible, we employ lookahead to circumvent this issue. When a constant competitive ratio is possible, we provide almost matching lower and upper bounds on the best achievable one.

## 4 The Orchestration problem, an applicative aspect of the multistage framework

Finally, we study the Orchestration problem. Given a target sound, a number of time steps, a database of sounds (corresponding to instruments, notes and other parameters) and a number of constraints regarding the instruments, the Orchestration problem consists in finding a sequence of solutions as close as possible to the target sound, according to a distance measure, and that does not change too much, i.e keeping the same instrument, keeping the same note with the same instrument... We study this problem using a LP-based approach and compare the results with an already impletemented genetic algorithm.

## References


[1] Evripidis Bampis, Bruno Escoffier, Michael Lampis, and Vangelis Th. Paschos. Multistage matchings. In *Scandinavian Symposium and Workshops on Algorithm Theory (SWAT)*, pages 7 :1–7 :13, 2018.

[2] Evripidis Bampis, Bruno Escoffier, and Sasa Mladenovic. Fair resource allocation over time. In *International Conference on Autonomous Agents and MultiAgent Systems (AAMAS)*, pages 766–773, 2018.

[3] David Eisenstat, Claire Mathieu, and Nicolas Schabanel. Facility location in evolving metrics. In Javier Esparza, Pierre Fraigniaud, Thore Husfeldt, and Elias Koutsoupias, editors, *Automata, Languages, and Programming - 41st International Colloquium, ICALP 2014, Copenhagen, Denmark, July 8-11, 2014, Proceedings, Part II*, volume 8573 of *Lecture Notes in Computer Science*, pages 459–470. Springer, 2014.

[4] Anupam Gupta, Kunal Talwar, and Udi Wieder. Changing bases : Multistage optimization for matroids and matchings. In Javier Esparza, Pierre Fraigniaud, Thore Husfeldt, and Elias Koutsoupias, editors, *Automata, Languages, and Programming - 41st International Colloquium, ICALP 2014, Copenhagen, Denmark, July 8-11, 2014, Proceedings, Part I*, volume 8572 of *Lecture Notes in Computer Science*, pages 563–575. Springer, 2014.