Complexity Results for Common Due Date Scheduling Problems with Interval Data and Minmax Regret Criterion

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Abstract

We consider the problem of scheduling independent jobs with a common due date on a single machine with the objective of maximizing the number of early jobs. The processing times are uncertain and take any value from a certain job dependent interval. For measuring the quality of an algorithm for that problem with imprecise data we use the concept of minimizing the maximum regret. We present complexity results and some dominance properties.

Keywords: scheduling, minmax regret, number of late jobs, approximation algorithms, robust optimization.

We are given a single machine and n independent jobs j with processing times p_j, $j = 1, \ldots, n$ and common date d. The objective is to maximize the number of early jobs. The corresponding objective function is $\sum_{j=1}^{n} \bar{U}_j$ where C_j is the completion time of job j and $\bar{U}_j$ is defined as $U_j = 1$ if $C_j \leq d$ and $U_j = 0$ if $C_j > d$. In the classical three field notation the problem is denoted as $1|d_j = d| \sum \bar{U}_j$. It can also be considered as a knapsack problem with capacity d, weights p_j and unit profits 1. For fixed processing times the problem can be easily solved in $O(n \log n)$ time by sorting the jobs according to increasing processing times. In this paper we assume that the processing times p_j are uncertain and take any value from a certain job dependent interval $[a_j, b_j]$. W.l.o.g., we assume that all input data, i.e., a_j, b_j, d, take only integer values. For any set of jobs J we denote by $p(J)$ the total processing time of the jobs in J. Note that we consider the problem with objective function of maximizing the number of early jobs $\sum_{j=1}^{n} \bar{U}_j$ instead of the equivalent problem with objective function of minimizing the number of late jobs $\sum_{j=1}^{n} U_j$ because the former is closely related to the knapsack problem. It can be easily checked that all our results also hold for the objective function of minimizing the number of late jobs. For measuring the quality of an algorithm for that problem with imprecise data we use the concept of *minimizing the maximum regret.* An assignment of specific values p_j from the intervals $[a_j, b_j]$ is called a *(processing time) scenario.* The lower interval bound a_j is also called *minimal processing time*, and the upper interval bound b_j *maximal processing time*, respectively.

Let Γ denote the set of all possible scenarios and Π the set of all permutations of the jobs $1, \ldots, n$. For any scenario $P = (p_1, \ldots, p_n)$ and any sequence X of the jobs the number of early jobs is denoted as $F(X, P)$. In this paper we will deal with the two problems *maximizing the regret* and *minimizing the maximum regret.* Our problems are illustrated by the two agents Alice and Bob. Alice selects a sequence X of jobs. The problem of Bob, *Bob's problem*, is defined for every

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feasible sequence X by Alice and it consists in selecting a sequence $Y = Y(X)$ and a scenario $P = P(X)$ such that the *regret* (of Alice) $R(X, Y, P) = F(Y, P) - F(X, P)$ is maximized. The value $Z(X) = \max_{Y \in \Pi, P \in \Gamma} R(X, Y, P)$ denotes the *maximum regret* for X. We call this problem also *Bob's problem.*

Alice has to select a sequence X which minimizes her maximum regret, i.e., $\min_{X \in \Pi} Z(X)$. We call this problem also *Alice's problem.*

The fact that for most of the problem under consideration polynomial time algorithms which deliver the optimal solution are difficult to find, stimulates the search of approximation algorithms that deliver solutions fairly close to the optimum.

Research has been interested in minmax regret scheduling problems. An important objective function which has been investigated is minimizing the weighted sum completion times, see e.g. the paper by Lebedev and Averbakh [7] for complexity results for this problem. A recent survey on minmax regret scheduling problems is due to Kasperski and Zieliński [6].

Our contributions. All our results are available in [5]. For totally ordered jobs, we proved that we could find an optimal sorting, by using some useful dominance rules. We presented a pseudopolynomial algorithm for Bob's problem, but could also show that there is no possible approximation for it with a performance ratio better than 1/2. Finally, it was proven that Alice's problem is NP-hard in the ordinary sense and that it does not admit an FPTAS.

For most of the examined minmax regret problems in the literature a change in the interval data has no influence on the feasibility of a solution. For example, in the minmax regret knapsack problem interval profits are investigated, but not interval weights [4]. Also most of results for the minmax regret selecting items problem are for interval weights. Thus, our problem seems particularly difficult since a small change of the processing times can change the solution feasibility.

References

[1] H. Aissi, M.A. Aloulou, M.Y. Kovalyov, Minimizing the number of late jobs on a single machine under due date uncertainty, Journal of Scheduling 14 (2011) 351-360.

[2] H. Aissi, C. Bazgan, D. Vanderpooten, Approximation of min-max and min-max regret versions of some combinatorial optimization problems, Discrete Optimization 179 (2007) 281–290.

[3] I. Averbakh, On the complexity of a class of combinatorial optimization problems with uncertainty, Mathematical Programming 90 (2001) 263-272.

[4] F. Furini, M. Iori, S. Martello, M. Yagiura, Heuristic and exact algorithms for the interval min-max regret knapsack problem, INFORMS Journal on Computing 27 (2015) 392–405.

[5] I. Kacem, H. Kellerer, Complexity Results for Common Due Date Scheduling Problems with Interval Data and Minmax Regret Criterion, Discrete Applied Mathematics (2018), DOI: https://doi.org/10.1016/j.dam.2018.09.026

[6] A. Kasperski, P. Zieliński, Minmax (regret) scheduling problems, in: Y.N. Sotskov, F. Werner (Eds.), Sequencing and scheduling with inaccurate data, Nova Science Publishers, 2014, pp. 159–210.

[7] V. Lebedev, I. Averbakh, Complexity of minimizing the total flow time with interval data and minmax regret criterion, Discrete Applied Mathematics 154 (2006) 2167-2177.