Linear time algorithms on mirror trees

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Given is an oriented graph $G = (X, E)$ where X and E denote the set of nodes and arcs of G, respectively. A linear ordering (arrangement) on X is defined as a one to one mapping from X to $\{1, \ldots, |X|\}$. We say that τ, a linear ordering on X, is compatible with graph G if, and only if, for any arc (x, y) in E, we have that $\tau(x) < \tau(y)$, also written as $x \ \tau \ y$ to indicate that x comes before y . We seek a linear ordering τ, defined on X, compatible with graph G, that minimizes the directed sum-cut cost function defined as follows:

$$f(G, \tau) = \sum_{u \in X} \max\{\tau(\Gamma^+(u))\} - \tau(u), \tag{1}$$

where $\Gamma^+(u)$ denotes the set of adjacent nodes (direct successors) of node u.

The directed sum-cut criterion was introduced in the context of memory management to model the sum of variable lifespans for a program (Bossart T., Munier-Kordon A., Sourd F. (2007): Memory management optimization problems for integrated circuit simulators, *Discrete Applied Mathematics*, 155, pp. 1795-1811).

The linear ordering problem with respect to criterion (1) is shown to be $\mathcal{NP}$-hard even in the case of bipartite graphs, and linearly solvable in the case of in-trees and out-trees (see the above paper). In this paper we discuss the problem of recognizing a mirror tree and the linear ordering problem on a mirror tree with respect to the directed sum-cut criterion. Let us note that, as we will see it in the next sections, a mirror tree is nothing else than a kind of combination of an intree and its outree counterpart, called also divide-and-conquer graph.

In this work, we first introduce the concept of mirror graphs and discuss the complexity of recognizing such graphs. We then restrict our study to mirror trees and exhibit two linear time algorithms. The first is about the recognition of such graphs. The second is about the linear arrangement problem with respect to the directed sum-cut criterion as in (1).

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