

# Airline schedule planning with itinerary based demand

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10 janvier 2020

## 1 Context

The aircraft schedule planning problem aims at choosing the set of flight legs operated by an airline to maximize the airline's revenue. An airline typically solves this problem when it wants to develop its offer. In that case, based on the demand observed on each market (origin-destination), the company will change the frequency and timing of the different flights operated. Currently, the optimization problem solved by airlines uses a very rough model for revenue evaluation: they assume that each flight will generate a given revenue, independently of the others.

Three phenomenons are not taken into account by such a model. First, the revenue is not generated by leg, but by customers booking itineraries (sequences of legs between an origin and a destination). Thus, when a customer buys an itinerary, it takes available seats in several legs at the same time. Supposing that the revenue is leg-based, with legs being independent of each other, is ignoring the value of having a network.

Secondly, the customers have nowadays the possibility to compare the different itineraries available through online flight comparators, and they choose the itinerary that best fits their needs. The way a customer makes his choice must be understood in order to estimate the revenue correctly.

Finally, a factor that greatly influences the revenue associated with a planning, is the evolution of itinerary prices over time. The company naturally wants to find the evolution of prices that guarantees the best possible revenue. This problem is called the revenue management problem. More precisely, this problem consists in finding the evolution of prices for a set of products (here the itineraries) that will maximize the revenue generated by the customer buying the products over time.

Taking into account those three aspects precisely and simultaneously in the planning optimization leads to an intractable problem. We propose a model for the schedule planning problem that takes into account the network interactions and customer behavior in the creation of the schedule. To model customer behavior we use a discrete choice model. We also take into account operational constraints that ensures that the airline can operate the schedule with its fleet of aircraft and the slots available at the airports. Some contributions in the literature have similar revenue models [1], but they do not take into account the operational constraints.

## 2 Revenue model and stochastic optimization problem

Our model is as follows: we assume that the company sells a certain amount of tickets for each itinerary, respecting the number of seats available in the different planes. The company sets the number of tickets it will sell on each itinerary, and those numbers are not updated as the tickets are sold.

Then we suppose that  $D$  customers arrive successively on each origin-destination, and that they choose the itinerary they purchase according to the discrete choice model. This leads to a

stochastic problem which is still intractable. To tackle this issue, we propose a fluid approximation of this model. The fluid approximation model is derived from the General Attraction Model (GAM) introduced by Gallego et al. [2] and can be expressed as a linear program.

We show that the stochastic optimization model converges in probability to its fluid approximation in the atomistic client limit. Similar results have been proved on other stochastic problem approximations [3, 4], but our setting requires to develop a new proof approach. We also prove that the stochastic problem solution converges exponentially quickly to the fluid approximation solution when the number of customers and the capacities scale up.

### 3 Benders decomposition for the fluid model

We propose a MILP that models the aircraft schedule planning problem with a fluid approximation of the logit model that rules the customer behavior. Given the number of itineraries on the network to manage (several millions), the size of the problem is very large. Current solvers cannot frontally solve this problem.

We therefore propose a Benders decomposition approach to tackle this problem. This decomposition enables us to split the problem in an aircraft schedule problem on the one hand and a discrete choice problem over a market on the other hand, both of them being manageable.

## Références

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