A two-stage robust approach for minimizing the weighted number of tardy jobs with profit uncertainty

Henri Lefebvre[1], François Clautiaux[2], Boris Detienne[2]

[1] DEI, University of Bologna, Italy
henri.lefebvre@unibo.it
[2] IMB, University of Bordeaux, Inria Bordeaux Sud-Ouest, France
{francois.clautiaux,boris.detienne}@math.u-bordeaux.fr

Mots-clés : *Single machine scheduling, Robust optimization, Exact method*

1 Introduction

We investigate a stochastic variant of the well-known $1|r_j|\sum w_j U_j$ problem, in which the jobs are subject to unexpected failure that incur additional costs. The decision maker is then allowed to take recourse actions such as outsourcing or spending more time on the jobs to fix them. We are interested in worst-case optimization, with polyhedral uncertainty set affecting the objective function.

In our problem, called *Two-Stage Robust Weighted Number of Tardy Jobs* (2SRWNTJ) in the sequel, an instance consists of a set of jobs $\mathcal{J}$, each of which is characterized by a release date r_j, a due date d_j, and a nominal processing time p_j. A weight w_j can be interpreted as the cost for executing the job, or as the opposite of the profit associated with the processing of the job. At first stage, *here-and-now* decisions are to select a subset of jobs $\mathcal{J}^* \subseteq \mathcal{J}$ to process. After that, a subset of the jobs can be affected by unexpected failures, those being governed by the uncertainty set $\Xi = \{\xi \in \mathbb{R}_+^{|\mathcal{J}|} | \xi_j \leq 1, \forall J_j \in \mathcal{J} \text{ and } \sum_{j|J_j \in \mathcal{J}} \xi_j \leq \Gamma\}$. The realization of alea $\xi \in \Xi$ determines a profit degradation for each job $J_j \in \mathcal{J}$ defined as $\delta_j(\xi) = \bar{\delta}_j \xi_j$, where $\bar{\delta}_j$ is the maximum additional cost linked to the job's failure. Input parameter Γ is, therefore, the largest number of jobs that can incur their maximum degradation. At second stage, *recourse* actions have to be taken. For each $\mathcal{J}^*$, one can choose (i) to keep the revealed profit ; (ii) to repair the job, adding τ_j time units to its processing time to recover its initial profit ; or (iii) to reject the job and pay a fixed outsourcing cost f_j. Finally, jobs in $\mathcal{J}^*$ that are not rejected must be scheduled so that they meet their time windows. The objective is to select a subset of jobs as well as the recourse actions that minimize the worst-case overall cost.

2 Related work

In [1], the authors study a variant of $1||\sum U_j$ where the processing times are uncertain. Given a discrete scenario-based uncertainty set, one has to determine an initial, feasible for nominal processing times, sequence of jobs. At second stage, once the scenario of actual processing times is revealed, the sequence must be made feasible for those actual processing times by rejecting some jobs. The objective is to minimize the expected cost of the repaired solution. Exact methods are proposed for this problem. Our study differs by the basic problem (we consider unequal release dates and weights), the nature of the uncertainty set (polyhedral vs. discrete), the uncertain data (objective vs. constraints) and the possible recourse actions.

Robustness is known to be a hard issue in scheduling. [4] and [5] show that even simple scheduling problems become $\mathcal{NP}$-hard as soon as the uncertainty set contains more than one scenario. A possible way to addess our problem is to use the so-called *finite adaptability* introduced by [6] for which MILP formulations have been presented in [7]. This heuristic approach consists in restricting the problem by determining, at first stage, a set of K recourse solutions, while the second stage is reduced to choosing the best of those for the revealed alea. On the one hand, for small values of K, this approach has the advantage to produce tractable problems. On the other hand, it may produce suboptimal solutions, since it restricts the number of recourse actions that can be performed.

3 Main contribution

The main contribution of our work is to propose the first exact method for this problem. It is based on a recent result of [3]. In particular, we first model the (2SRWNTJ) problem as a standard two-stage robust problem with second-stage integer decisions by extending some results found by [2] for the deterministic $1|r_j|\sum w_j U_j$ problem. We then show how the methodology introduced by [3] can be used to reformulate our problem as a MILP formulation. The application of the aforementioned method is motivated by some interesting features of our initial formulation. Namely, a polyhedral uncertainty set and interdiction linking constraints between the first stage and second stage (i.e., constraints of the form $\alpha \leq \beta$ where α and β are binary decisional vectors).

Because the obtained formulation is of exponential size, we solve it with a branch-and-price algorithm. Finally, we compare our approach with the previously mentioned finite adaptability heuristic. We show that our exact approach outperforms it for the most difficult instances, namely, those with a larger number of jobs. In particular, we are able to solve to optimality all 20-jobs instances of our test bed within one hour, and 85% of the 25-jobs instances within the same time limit while the *finite adaptability* approach fails at solving some 10 jobs-instances, even by assuming to know the optimal parameter K^*. Finally, it solves less than 17% of the instances for which $K^* \geq 2$ and $|\mathcal{J}| = 25$.

Références

[1] van den Akker, Marjan and Hoogeveen, Han and Stoef, Judith (2018) *Combining two-stage stochastic programming and recoverable robustness to minimize the number of late jobs in the case of uncertain processing times.* Journal of Scheduling 21(6), 877-891

[2] Detienne, Boris (2014) *A mixed integer linear programming approach to minimize the number of late jobs with and without machine availability constraints.* European Journal of Operational Research 235, 540-552

[3] Arslan, Ayse Nur, Detienne Boris (2018) *Reformulation approaches for a two-stage robust knapsack problem* International Workshop on robust optimization 2018, Avignon

[4] Aloulou, Mohamed Ali and Della Croce, Federico *Complexity of Single Machine Scheduling Problems Under Scenario-based Uncertainty* Oper. Res. Lett. 36(3), 338-342

[5] Yang, Jian and Yu, Gang *On the Robust Single Machine Scheduling Problem* Journal of Combinatorial Optimization, 6(1), 17-33

[6] D. Bertsimas and C. Caramanis *Finite Adaptability in Multistage Linear Optimization* IEEE Transactions on Automatic Control, 55(12), 2751-2766

[7] Hanasusanto, Grani A. and Kuhn, Daniel and Wiesemann, Wolfram *K-Adaptability in Two-Stage Robust Binary Programming* Operations Research, 63(4), 877-891