Non necessarily continuous piecewise linear approximation with a performance guarantee : application to mixed integer optimization

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We present an efficient algorithm able to over-estimate/under-estimate/approximate any arbitrary univariate nonlinear derivable function by a non necessarily continuous piecewise linear function that minimizes the number of linear segments with a guaranteed tolerance. The algorithm is based on the piecewise linear bounding method recently proposed by [Ngueveu 2018]. The two main methodological contributions are :

— a generalization of the approach to a larger class of tolerance types than the absolute and relative tolerances from Ngueveu
— a reformulation technique allowing any approximation problem of convex or concave function with any tolerance type that preserves concavity, to be reduced to fitting a piecewise linear function within a bounded corridor.

An efficient implementation of the piecewise linear function fitting has been made available in a JULIA library called LinA. The resulting software can be used to solve certain classes of MINLP problems with linear constraints and a non-linear objective function.

Computational results on nonlinear functions approximation benchmark [Rebennack and Krasko, 2019] and on network design problems with congestion [Paraskevopoulos et al, 2016] will be presented.

Références

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