Production planning in an industrial symbiosis context

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1 Introduction

Industrial symbiosis represents a sustainable way of sharing resources and converting unavoidable production residues into useful and high added-value products. One of the most common beneficial form of symbiotic industrial production is the process by which, by-products of a production unit become the raw material for another one. By definition, by-products are lawful undesirable production outputs, whose further use is economically and environmentally sustainable.

Among the abundant number of industrial joint production systems studied in the literature [4], little has been focused on by-products management. It is worthwhile to mention the work of Sridhar et al. [3], who studied a generic non-linear continuous-time production problem dealing with by-products. Under discrete time settings, Suzanne et al. [5] proposed a generic lot-sizing problem with by-product inventory capacities.

Depending on the industrial sector and the production process, the nature of the by-product can differ: *unstorable* (e.g. water steam, heat) or *storable* (e.g. water, sludge). Note that in some particular cases, the inventory capacity of the by-product can be limited for technical reasons (e.g. sludge synergy in the Kalundborg symbiosis in Denmark).

Given the circular economy concerns and industrial needs, let us investigate a single-item lot-sizing problem arisen by the industrial symbiosis between two production units (PU1 and PU2) with disposal and purchasing options, subject to a limited by-product inventory capacity. This form of production can be related to the well-studied bi-level production planning problem, where a first-level entity has to determine its production plan and that of the second-level entity [1, 2]. In this paper, the levels corresponds to the considered production units PU1 and PU2, and are linked via the by-product created by PU1 and used subsequently as a raw material by PU2.

2 Problem statement

Consider an industrial symbiosis single-item lot-sizing problem (ULS-IS) between two production units PU1 and PU2. Each production unit produces a different product in order to meet its own deterministic demand. PU1 generates a by-product at the same time as its main product. The generated by-product can be: disposed of, sent to PU2 for a unitary cost lower than the unitary disposal cost, or stored with a unitary holding cost. Note that, the by-product inventory is limited by a constant capacity. To supply its production with raw materials, PU2 can choose either to use the by-product generated by PU1 or to order at an

external supplier. The unitary cost of buying the by-product is lower than the purchasing cost of the raw material. In addition to the aforementioned costs, the classical costs related to the lot-sizing problems are included in the objective function for each production unit, namely: fixed setup costs, unitary production costs and unitary inventory holding costs of the two main products. The objective function aims at minimizing the sum of all costs of both PU1 and PU2 occurring over the whole planning horizon.

3 Solution approach and performance evaluation

We show that the ULS-IS problem is NP-Hard by reduction from the capacitated lot-sizing problem. To solve the ULS-IS problem, we propose a Lagrangian decomposition algorithm. In this sense, variables corresponding to the by-product flows are duplicated. This operation, coupled with the relaxation of capacity constraints, leads to the problem decomposition into two subproblems SP1 and SP2, that can be solve using dynamic programming algorithms running in $O(T \log T)$. In order to construct feasible solutions from the Lagrangian solution obtained by solving optimally SP1 and SP2, we propose the following Lagrangian heuristic, which operates in two phases:

- **Smoothing phase:** This phase constructs a feasible solution of the ULS-IS problem by, at each period: **(i)** matching the quantity of by-product transported between PU1 and PU2, and **(ii)** checking the stored quantity of the by-product to fulfill capacity constraints.
- **Improvement phase:** This phase consists in moving production quantities from a period to another one with a view to reducing the disposal and purchasing quantities. Moves can be done forward or backward.

To evaluate the proposed solution approach, numerical experiments are conducted on heterogeneous instances of different size. A special interest is carried on the particular case where the by-product is unstorable. To infer the managerial implications of the industrial symbiosis and to highlight the mutual benefits procured by an equitable collaboration between the considered entities, two power struggles are studied, namely with centralized and decentralized decisions.

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