# Solving Techniques for a Demand-based Revenue Maximization Model

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This document aims at providing an overview of the internship I completed within the Transport and Mobility laboratory (TRANSP-OR) at *École Polytechnique Fédérale de Lausanne* (EPFL). This internship constitutes the final semester of the master's degree in Microelectronics and Computer Science (ISMIN program) of the French engineering high school *École Nationale Supérieure des Mines de Saint-Étienne* (ENSM-SE). Its goal is to investigate and compare several solving techniques for a demand-based revenue maximization model.

# 1 Problem Overview

## 1.1 Demand-based Revenue Maximization Problem

Consider an operator that aims at finding the best pricing strategy to maximize its revenue. We assume that this operator sells services $i \in C$ (except an opt-out option $i = 0$) to a population of $N$ customers, at a certain price $p_{i,n} \in [a_i; b_i]$. These prices need to be decided as they affect the choices of customers (and thereby the revenue). The corresponding MILP formulation, denoted $(P)$, has been introduced in **Pacheco2019**.

In discrete choice modeling, each individual $n$ associates a score to alternative $i$ through a utility $U_{i,n}$, which is a function of the price $p_{i,n}$ along with an error term. For the sake of integration into MILP, this function is required be linear function in the decision variables $p_{i,n}$, and its probabilistic part is addressed by generating $R$ scenarios. At the end, each customer opts for the alternative maximizing its utility function.

## 1.2 Lagrangian decomposition of the Model

The model turns out to be computationally expensive for instances having large sizes. The idea is to apply Lagrangian decomposition to the problem in order to obtain $N \cdot R$ subproblems that can be solved more easily with respect to $(P)$. For that purpose, each price $p_{i,n}$ is split into $R$ variables $p_{i,n,r}$ while imposing equality between them. These equality constraints are then relaxed and their violation are penalized with multipliers in the objective function.

Each Lagrangian subproblems are solved to optimality, and a feasible solution is constructed to $(P)$ using $p_{i,n,r}$. This Lagrangian-based heuristic provides a lower and upper bound to $(P)$.

# 2 Outline of the Presented Work

## 2.1 Subgradient Optimization Method

In order to update the Lagrangian multipliers, the Subgradient Optimization Method is considered. This method is also compared to three of its variants summarized in **Guta2003**: the Deflected, Conditional and Hybrid Subgradients. They are designed to speed up the convergence by correcting the subgradient direction.

As outcome, no significant differences are noticed between the approaches, even with a sensitivity analysis and a study of two step strategies. For the best configuration, the upper and lower bounds remain quite poor, with a gap to the optimal over 26% and 7%, respectively. Besides, the CPLEX solver clearly proves to be more efficient than the subgradient approach.

## 2.2 Improvement Strategies

Various enhancements approaches are tested: object oriented approach, preprocessing strategies, subgradient adjustments... The most beneficial one is the grouping of the Lagragian subproblem, that allows to overcome the too strong decomposition initialy performed. The resulting formulation leads to a gap to the optimal below 1% for the lower bound and 6% for the upper one within a reasonable computational time.

## 2.3 Direct Heuristics

Nevertheless, a quite simple heuristic method for $(P)$ proves to outperform the two decomposition techniques. Based on an proven optimality criterion, a taylor-made algorithm provides an average gap of 1.88% with a solving time exponentially decreasing compared to CPLEX one. A less competitive heuristic is also designed to solve $(P)$ to its optimal for $|C| < 4$.

# 3 Future research directions

Several avenues are worth considering. A comparison of the heuristics has to be conducted with state-of-the-art (meta)heuristics, just like the subgradient method. If industrial applications are considered, all the decompositions can be parallelized in order to improve solving efficiency. From a research perspective, the assessment of theses methods to the extension of $(P)$ to the capacitated case with one only price $p_i$ already shows promising results.

# References

[1] Meritxell Pacheco (2019) A Lagrangian relaxation technique for the demand-based revenue maximization model *Technical Report TRANSP-OR*, Ecole Polytechnique Fédérale de Lausanne.

[2] Berhanu Guta (2003) Subgradient Optimization Methods in Integer Programming with an Application to a Radiation Therapy Problem PhD thesis, Universität Kaiserslautern.