On the complexity of the crew assignment problem

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1 Introduction

The airline crew scheduling problem is one of the most important problems in the airline planning process because the total crew cost (salaries, benefits, and expenses) is considered, next to fuel cost, the largest single cost of an airline company [1]. For large fleets, this problem is decomposed into a crew pairing problem and a crew assignment problem, both of which are solved sequentially. The first consists of generating a set of least-cost crew pairings (sequences of flights starting and ending at the same crew base) that cover all flights. The second aims at finding monthly schedules (sequences of pairings) for crew members that cover all pairings previously built. Pairing and schedule construction must respect all safety an collective agreement rules. In this work, we focus on the crew assignment problem. For an extensive review, see [6, 2, 3].

The crew assignment problem is generally considered a difficult optimization problem (NP-hard) since it can be formulated as a set partitioning problem or as a set covering problem [6]. In the present paper, we considered a sub-problem that has been encountered at Air France. The inputs of the problem consist of a set $I = \{I_1, \ldots, I_n\}$ of n activities to be assigned to a set of crew members $C = \{C_1, \ldots, C_m\}$, where each activity is represented by an interval $I_j = [l_j, r_j] \in I$ with fixed starting time l_j and fixed ending time r_j and each I_j can be covered by one crew member of C. The objective is to assign all the activities to a minimum number of crews in a Horizon (H) of one month with respect to a Rest Period (RP) of 13 days that contains 7 consecutive days called Rest Bloc (RB).

We first present a graph representation of the problem. Based on this graph, we study the complexity of the problem when restricted to one crew member, then we consider the general case of a set of crew members.

2 Graph representation

We associate to the set of activities a weighted directed graph $G = (V, A, W, U)$ as follows :

— $V = V_1 \bigcup \{s, p\}$, where the vertices of V_1 represent the activities of I, s is the source with $l_s = r_s = 0$ and p is the sink with $l_p = r_p = R$ (R is the end of the month).

— $A = A_1 \bigcup A_2 \bigcup A_3$, where $A_1 = \{(s, v_j) : v_j \in V_1\}$, $A_2 = \{(v_j, p) : v_j \in V_1\}$ and $A_3 = \{(v_i, v_j) : v_i, v_j \in V_1 \text{ and } r_i <= l_j\}$.

— To each vertex $v_j \in V$ is associated a weight $w_j \in W$, where $w_j = r_j - l_j$.

— To each arc $(v_i, v_j) \in A$ is associated a length $u_{ij} \in U$, where $u_{ij} = l_j - r_i$.

It is clear that the graph G is acyclic and transitive.

3 Contribution

We first consider the case of one crew member, the problem consists of maximizing the working time of the crew member with respect to RP and RB (the problem is denoted $1CA$). We show the following result :

Proposition 1 *$1CA$ can be solved in polynomial time by dynamic programming on the graph G.*

However, when H is not fixed, the problem becomes as NP-hard as the subset sum problem [5] even if the set of activities form a transitive closure and the elapsed time between each pair of activities includes a RB which is a constant.

Let A' be a subset of A containing the arcs with length greater than or equal to 7 days. The general problem (denoted mCA) can be seen as the partition of G into a Minimum number of vertex-disjoint Paths that Cover (denoted MPC [5]) all the vertices of G such that each path has a total weight of at most 18 (sum of the weights of the vertices) and contains at least one edge from A'. The following result is obtained by using the results of Lenstra [7] and Eisenbrand [4] on integer programming with a fixed number of variables and constraints.

Theorem 1 *mCA can be solved in polynomial time.*

However, when G is not transitive, the following variant of the MPC problem is NP-hard in the strong sense (by a reduction from the 3-dimensional matching problem [5]) for a subset A' of A satisfying : if $(v_i, v_j) \in A'$ and (v_k, v_l) includes (v_i, v_j), then $(v_k, v_l) \in A'$. We denote the directed acyclic graph satisfying this condition by G'.

Theorem 2 *The MPC problem on G' such that each path includes at least one edge from $A' \subseteq A$ is NP-hard in the strong sense.*

Now, when H is not fixed, the problem becomes as hard as the 2-partition problem [5] even if the set of activities form a transitive closure, the elapsed time between each pair of activities includes a RB which is a constant and the working time is half the duration of all activities.

4 Perspectives

An ongoing work is devoted to the construction of algorithms to build personalized schedules with the goal of maximizing the preferences of each crew member.

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